Theory

(1) Q.No.- State and Prove Lagrange's mean value theorem

Here I am going answers my Lagrange's mean value

Ans.- **Statement** :- If a function f is

(i) Continuous in closed interval $[a, b]$

(ii) differentiable in open interval $]a, b[$, then there exists at least one point c of the open interval $]a, b[$, such that

$$\frac{f(b) - f(a)}{b - a} = f'(c)$$

Proof :- To prove this theorem let us suppose a function F defined by

$$F(x) = f(x) + Ax$$

where A is a constant to be determined such that $F(b) = F(a)$

Thus given, $f(a) + Aa = f(b) + Ab$

$$Aa - Ab = f(b) - f(a)$$

$$-A(b-a) = f(b) - f(a)$$

$$\therefore -A = \frac{f(b) - f(a)}{b - a} \quad \text{——— (1)}$$

Now, f is differentiable in $]a, b[$ and x is diff. in any interval.

$$\because F(x) = f(x) + Ax$$

Hence, $F(x)$ is differentiable in $]a, b[$

Hence, all the conditions are satisfied for the function F. Hence there exists at least one point c of the open interval $]a, b[$ such that $F'(c) = 0$

$$\therefore F'(x) = f'(x) + A$$

$$F'(c) = f'(c) + A$$

$$\because F'(c) = 0$$

$$\therefore f'(c) + A = 0$$

from 1 & 2

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

It can also be put in another form by putting $b - a = h$, $c = a + \theta h$

where $0 < \theta < 1$

Hence, the above theorm can be written as,

$$f(a+h) - f(a) = hf'(a + \theta h)$$

$$(0 < \theta < 1)$$

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① Q.No→ State and prove Cauchy's mean value theorem

Ans.→ Statement:—

If two function f and g are

(i) Continuous in closed interval $[a, b]$

(ii) differentiable in the open interval $]a, b[$

(iii) ~~$f'(x) = 0$ for any point of the open interval~~ $]a, b[$ ~~then~~ there exists at least one value c ~~in the open interval~~ $\in]a, b[$ such that,

$$\frac{g(b) - g(a)}{f(\cancel{a}b) - f(a)} = \frac{g'(c)}{f'(c)} \quad a < c < b$$

Proof:- To prove this theorem, Let us Consider a function h given by

$$h(x) = [f(b) - f(a)]g(x) - [g(b) - g(a)]f(x)$$

for all $x \in [a, b]$

Then from Condition (i) & (ii) h is Continuous on $[a, b]$ and differentiable in $]a, b[$

$$h(a) = [f(b) - f(a)]g(a) - [g(b) - g(a)]f(a)$$

$$= f(b)g(a) - \cancel{f(a)g(a)} - g(b)f(a) + \cancel{g(a)f(a)}$$

$$h(a) = f(b)g(a) - g(b)f(a) = h(b)$$

Hence, by Rolle's theorem there exists a point $c \in]a, b[$

Such that $h'(c) = 0$

Now, $h'(x) = [f(b) - f(a)]g'(x) - [g(b) - g(a)]f'(x)$

$\because f(b) - f(a)$ &

$g(b) - g(a)$ are Const:-

$$\therefore h'(c) = [f(b) - f(a)]g'(c) - [g(b) - g(a)]f'(c)$$

$\therefore h'(c) = 0$

$\therefore [f(b) - f(a)] g'(c) - [g(b) - g(a)] f'(c) = 0$

or, $[f(b) - f(a)] g'(c) = [g(b) - g(a)] f'(c) = 0$

or, $$\frac{f(b) - f(a)}{g(b) - g(a)} = \frac{f'(c)}{g'(c)}$$

provided $g'(c) \neq 0$

It can also put in another form if f and g are continuous on $[a, a+h]$, differentiable on $]a, a+h[$ and $g'(x) \neq 0$ for any $x \in]a, a+h[$ then there exist one no. θ betn 0 & 1

Such that,

$$\frac{f(a+h) - f(a)}{g(a+h) - g(a)} = \frac{f'(a+\theta h)}{g'(a+\theta h)} \quad \theta \in]0, 1[$$